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6. AUTHOR(S)

Athanassios Tzavaras

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7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

University of Wisconsin-Madison
750 University Avenue
Madison, WI 53706

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13. ABSTRACT *(Maximum 200 words)*

In the framework of continuum mechanics the motion of deforming bodies is described by systems of nonlinear evolution equations, that arise by combining balance laws with constitutive relations characterizing the material response. The nonlinear character of the material response often induces a destabilizing mechanism that competes with dissipative mechanisms, such as viscosity or thermal diffusion. As a result of the competition coherent structures may appear, which at some level of modeling manifest themselves as singularities in the solutions of the corresponding model. These structures are diverse in nature, ranging from shock waves to shear bands to propagating phase boundaries, and no common theory can encompass all of them at present. Various instances of such phenomena were studied as part of the project. Specifically: (a) A class of test problems intended to test a thermoplastic instability mechanism for shear band formation at high strain rates. (b) Dynamics of spurt phenomena in viscoelastic flows. (c) Self-similar viscous and hydrodynamic limits for the equations of isentropic gas dynamics and the Broadwell system.

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Final Technical Report to DAAL03-88-K-0185

Systems of Evolution Equations in Thermomechanical Processes

Athanasios E. Tzavaras, University of Wisconsin-Madison

Research Description: In the framework of continuum mechanics the motion of deforming bodies is described by systems of nonlinear evolution equations, that arise by combining balance laws with constitutive relations characterizing the material response. The nonlinear character of the material response often induces a destabilizing mechanism that competes with dissipative mechanisms, such as viscosity or thermal diffusion. As a result of the competition coherent structures may appear, which at some level of modeling manifest themselves as singularities in the solutions of the corresponding model. These structures are diverse in nature, ranging from shock waves to shear bands to propagating phase boundaries, and no common theory can encompass all of them at present. Various instances of such phenomena were studied as part of this project:

Shear Bands. One of the most striking manifestations of instability in solid mechanics is the localization of shear strain into narrow bands during high speed, plastic deformations of metals. Shear bands are structures with a sharp transition layer in the displacement field and a corresponding concentration in strain. As they often precede rupture their understanding is critical for the improvement of materials, and their study has attracted a lot of attention from diverse points of view. The objective of this program is to understand a thermoplastic instability mechanism proposed for the explanation of shear band formation at high strain rates. According to this scenario, two main factors are competing during shear band initiation: First, effective strain-softening response, resulting as the net outcome of the influence of thermal softening on the (under isothermal conditions) strain-hardening response of metals. This is a destabilizing factor having the tendency to amplify nonhomogenuities. And second, strain-rate hardening which has the opposite effect of diffusing nonhomogenuities. To understand this competition various test deformations were considered. They lead to systems of nonlinear partial differential equations,

which are studied by means of nonlinear analysis techniques. The techniques include the use of comparison estimates, the existence of invariant regions and exploiting scale-invariance properties that specific models are equipped with. Some of the main findings are summarized below (for details see the technical description part of the report).

To assess the contributions of thermal softening, strain hardening and strain-rate sensitivity, a shearing deformation caused by prescribed tractions has been analyzed for a model based on a power law and for which explicit thermal effects are taken into account. It turns out that the parameter space formed by the powers can be decomposed into three distinct regions across which the behavior of the corresponding solutions changes drastically: from blow up, to weak instability, to asymptotic stability. The first two regions are associated with the development and evolution of spatial nonhomogenuities in the strain, in the form of shear bands, and are characterized by a collapse of the ability of the material to diffuse the applied stresses [1, 3].

To understand the mechanics of development and propagation of shear bands, a shearing deformation, caused by steady boundary shearing, of a strain-softening, rate-dependent material has been analyzed. A linearized stability analysis of the time-dependent linearized equations characterizes the necessary degree of strain softening to produce growing perturbations of the uniform shearing solutions. In a result for the nonlinear problem, the evolution of an initial large concentration of strain is monitored and shown to produce a fully developed shear band at large times; outside the band the material unloads [2, 4].

Self-Similar Limits. The invariance of hyperbolic systems of conservation laws under dilations of coordinates is a key property underlying much of the current theory. Viscous perturbations introduce an additional parabolic scale, and this invariance is lost. At present it is not known how these two scales interact in the context of viscous limits for the Riemann problem. One illuminating step in this direction is to study artificial regularizations, rigged so as to preserve the invariance under dilatations of coordinates. It leads to the study of singular perturbations for non-autonomous boundary-value problems, and the limiting process involves variation estimates. This approach was tested for the equations of isentropic gas dynamics in Eulerian coordinates [7], and is currently under study for larger systems. Recently, a similar idea was employed for the case of the Broadwell system

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with Riemann data. The system is modified so as to incorporate the invariance under dilations of the limiting conservation laws, the resulting self-similar solutions are studied, and the fluid-dynamic limit is carried out [8].

Viscoelasticity. A simple model problem was studied intended to test the following conjecture: That lack of monotonicity in the steady shear stress versus steady shear strain-rate leads to non-Newtonian response, manifested by the occurrence of spurt in viscometric flows of certain pclymer solutions. It was shown that a class of discontinuous steady states (associated with spurt) are asymptotically stable [5].

Technical Description A technical description of the projects supported by the U.S. Army Research Office under Grant DAAL03-88-K-0185 follows. We include studies that are in the preparation process and which (when published) will acknowledge support from this Grant.

Studies of the phenomenon of shear band formation at high strain rates

[1] *Strain softening in viscoelasticity of the rate type* J. Integral Equations Appl. **3** (1991), 195-238.

In this article we study the behavior of solutions of the system of partial differential equations

$$\begin{aligned} v_t &= \sigma_x \\ u_t &= v_x \end{aligned} \tag{1}$$

$$\sigma = \tau(u) v_x^n \tag{2}$$

where $\tau(u)$ satisfies $\tau(u) > 0$ and $\tau'(u) < 0$. The equations describe the plastic shearing of a plate of unit thickness. In this context, v stands for the shearing velocity, u for the plastic strain and σ for the shear stress. Our objective was to use this model as a paradigm, in order to analyze the competition between the destabilizing effect of strain softening versus the stabilizing influence of strain-rate sensitivity, the strength of which is measured by the (typically small) parameter n.

A general existence and continuation theory of classical solutions, for problems fitting under the structure $(1-2)$, is carried out. Subsequently, the time-evolution of the solutions corresponding to two types of loading is studied:

Prescribed tractions $\sigma(0,t) = \sigma(1,t) = 1$: A characterization of global existence or blow up of solutions in terms of convergence properties of the integral $\int_1^\infty \tau(\xi)^{\frac{1}{n}} d\xi$ is first established. Then the response of solutions is discussed under various hypotheses for the function $\tau(u)$. We present the result for a power law $\tau(u) = 1/u^m$ with $m > 0$. The parameter region can be decomposed into three distinct subregions accross which the response changes drastically. For $0 < m/n < 1/2$, solutions are asymptotically attracted to a stable time-dependent state; for $1/2 \leq m/n \leq 1$, this state loses its attracting property; and for $m/n > 1$, solutions blow up in finite time. This change of behavior is associated with inability of the material to diffuse the applied stresses, and developement of spatial nonuniformities in the strain (associated with shear band formation) at the boundaries.

Prescribed velocities $v(0,t) = 0$, $v(1,t) = 1$: We consider a power law, $\tau(u) = 1/u^m$, and use scaling invariance properties together with the existence of invariant regions for an associated parabolic system to discuss the stability of a special class of solutions, describing shearing. It turns out that, for $0 < m/n < 1$, the uniform shearing solutions are stable and every solution is asymptotically attracted as $t \to \infty$ to one of them.

[2] *Shear strain localization in plastic deformations* in "Shock Induced Transitions and Phase Structures in General Media", (R. Fosdick, E. Dunn and M. Slemrod, eds), IMA Volumes in Mathematics and its Applications, Springer Verlag, 1991 (to appear).

In this article the system $(1-2)$ above is discussed. The first part is a summary of the results presented in [1].

In the second part, we take up a power law $\sigma = (1/u^m)v_x^n$ for shearing caused by prescribed velocities and consider the parameter region $m/n > 1$ (complementary to the case discussed in [1]). It is shown that, for initial data corresponding to a linear velocity profile and a concentration of strain, either the corresponding strain blows up, or as $t \to \infty$

$$v(x,t) = \begin{cases} O(t^{-1}) & x \in [0, y-\delta] \\ 1 + O(t^{-1}) & x \in [y+\delta, 1] \end{cases}$$

and $u(x,t)$ converges outside the region $[y-\delta, y+\delta]$ to a limiting configuration. Thus the material unloads outside the band.

[3] *On adiabatic shear bands* (in preparation).

This manuscript is still in the preparation process. Here we present some results available at the time of the writing.

Our goal in this article is to extend the understanding of the shear band formation problem obtained in [1,2] for cases that explicit thermal effects are taken into account. We consider the model

$$
\begin{aligned}
v_t &= \sigma_x \\
u_t &= v_x \qquad (3)\\
\theta_t &= \sigma v_x
\end{aligned}
$$

$$
\sigma = \mu(\theta, u) v_x^n \qquad (4)
$$

with boundary conditions: either prescribed tractions or prescribed velocities. The function μ is assumed decreasing in θ, and n is positive measuring the degree of strain-rate sensitivity. Regarding the dependence of μ in u both strain hardening ($\mu_u > 0$) and strain softening ($\mu_u < 0$) are admitted as possibilities, but an assumption that guarantees that thermal softening dominates over strain hardening is instead imposed. This resulting system is a model to assess the interplay of thermal softening, strain hardening and strain-rate sensitivity on the response of plastic flows.

For loading effected by prescribed tractions, a characterization of global existence or blow-up in terms of the behavior of $\mu(\theta, u)$ is presented. Then the behavior of solutions is analyzed. To illustrate, consider the case of a power law $\sigma = \theta^\nu u^m u_t^n$, with parameters $\nu < 0$, m and $n > 0$. It turns out that the parameter region can be decomposed into three subregions: (a) $\nu + m < -n$, (b) $-n < \nu + m < -n/2$ and (c) $-n/2 < \nu + m < 0$. In region (c) the solutions exhibit a stable response, in region (b) they develop spatial nonuniformities and in region (c) they blow up. This generalizes the analysis in [1].

In the case of prescribed velocities, the case when strain dependence is absent is considered

$$
v_t = (\mu(\theta) v_x^n)_x
$$

$$\theta_t = \mu(\theta) v_x^{n+1}$$

where $\mu(\theta)$ is decreasing to 0. It is shown that, for certain classes of initial data, either the solution blows up or the uniform shearing solution is unstable.

[4] *Nonlinear analysis techniques for shear band formation at high strain rates*, Appl. Mech. Reviews (3) **45** (1992), 82-94.

This article presents a review of the results of [1-3] emphasizing the ramifications on the mechanics aspects of the shear band formation problem. In addition, a linearized stability analysis for the uniform shearing solutions in the context of $(1-2)$, taking account of the time dependence of the uniform shearing solutions, is presented here.

Studies related to viscoelasticity

[5] *Stability of discontinuous steady states in shearing motions of a non-Newtonian fluid* (with J.A. Nohel and R.L. Pego), Proc. Roy. Soc. Edinburgh **115A** (1990), 39-59.

We study the nonlinear stability of discontinuous steady states of the following model initial-boundary value problem in one-space dimension, describing incompressible, isothermal shear flow driven by a constant pressure gradient

$$\begin{aligned} v_t &= S_x = (\sigma + v_x + fx)_x \\ \sigma_t + \sigma &= g(v_x) \end{aligned} \tag{5}$$

with boundary conditions $S(0,t) = 0$, $v(1,t) = 1$. The non-Newtonian contribution to the stress satisfies a differential constitutive law. The problem shares certain key features with models proposed to explain the occurrence of "spurt" phenomena in non-Newtonian flows. This system admits steady state solutions $(\bar{v}(x), \bar{\sigma}(x))$ satisfying

$$g(\bar{v}_x) + \bar{v}_x + fx = 0 \qquad \bar{\sigma} = g(\bar{v}_x)$$

In the case of interest the function $w(\xi) ;= g(\xi) + \xi$ is non-monotone, and as a result the problem admits multiple steady states, leading to velocity profiles with kinks. We show that every solution tends to a steady state as $t \to \infty$. Moreover, we show that the steady

states for which $\bar{v}_x(x)$ takes values in the monotone increasing parts of the graph of $w(\xi)$ are stable.

[6] *Nonlinear stability in non-Newtonian flows*, (with J.A. Nohel and R.L. Pego), in "Multidimensional Hyperbolic Problems and Computations" (J. Glimm and A. Majda eds.), pp. 251-260, IMA Volumes in Mathematics and its Applications, Vol. 29, Springer Verlag, New York, 1991.

This is a conference proceedings report outlining the results in [6].

Studies related to self-similar limits

[7] *A limiting viscosity approach for the Riemann problem in isentropic gas dynamics*, (with M. Slemrod), Indiana Univ. Math. J. **38** (1989), 1047-1074.

We consider the equations of isentropic gas dynamics in Eulerian coordinates

$$\begin{aligned} \rho_t + (\rho u)_x &= 0 \\ (\rho u)_t + (\rho u^2 + p(\rho))_x &= 0 \end{aligned} \tag{6}$$

with $p'(\rho) > 0$ and Riemann data. We construct solutions of this problem as limits of solutions of a "viscosity" regularized problem that is rigged so as to preserve the invariance of (6) under dilations of the independent variables. The limiting procedure is based on variation estimates for the solutions of an associated system of ordinary differential equations. The solutions thus constructed may contain vacuum regions.

[8] *Self-similar fluid dynamic limits for a modified Broadwell system* (with M. Slemrod), (preprint).

We study self-similar hydrodynamic limits for a modified Broadwell model, which once transformed to self-similar variables reads:

$$\begin{aligned} (-\xi + 1) f_1' &= \frac{1}{\varepsilon}(f_3^2 - f_1 f_2) \\ (-\xi - 1) f_2' &= \frac{1}{\varepsilon}(f_3^2 - f_1 f_2) \\ -\xi f_3' &= \frac{1}{2\varepsilon}(f_1 f_2 - f_3^2) \\ f_i(\pm\infty) &= f_{i,\pm}, i = 1, 2, 3 \end{aligned} \tag{B}$$

with data corresponding to "Maxwellians", that is $f_{3,\pm}^2 - f_{1,\pm} f_{2,\pm} = 0$. The variation estimates as $\varepsilon \searrow 0$ are based on the observation that $Q = f_3^2 - f_1 f_2$ can only vanish at the singular points $\xi = -1, 0, +1$ (corresponding to characteristic directions for the Broadwell model). Thus the f_i's can change monotonicity only at these points. This is the main observation that allows the hydrodynamic limit $\varepsilon \searrow 0$ to be carried out for any data.

Because of the singularities at $-1, +1$ and 0, the existence of solutions for (B) requires some work. The approach used is based on linearization, uses the Frobenius theory for systems, and is restricted for data that are close to a Maxwellian.